International Journal of Theoretical Physics, Vol. 31, No. 12, 1992

Cosmological Compactification in Kaluza–Klein Model and Time-Dependent Cosmological Term

S. K. Srivastava[1]

Received January 21, 1992

Einstein's equations for the generalized $(4+D)$-dimensional Robertson–Walker model are solved taking the conformally invariant action for the matter field. Compactification of this model is discussed and the compactification time/compactification mass scale for different values of D is calculated. The resulting 4-dimensional action for gravity is obtained. It is found that a time-dependent cosmological constant is induced which is very large when the cosmic time is small and very small when the cosmic time is large.

In the context of the unification of gravity with fundamental forces of elementary particles, Kaluza (1921)–Klein (1926) theory and its generalization to higher dimensions is very attractive. The central problem in these theories is how the internal manifold (space with extra dimensions) compactifies to the Planck size. The topology of the space-time is supposed to be $\mathbb{R}$ (time) $\times M^3$ (ordinary 3-dimensional space) and S^D (D-dimensional internal manifold) with S^D compact and $\mathbb{R} \times M^3$ paracompact. Many authors have used this idea, but in most of the papers (Miller, 1977; Carter, 1977; Appelquist *et al.*, 1987) the scale factors associated with S^D are time-independent. This approach completely ignores the dynamical contribution of the internal manifold. It seems natural to study compactification and other manifestations of the internal manifold keeping its dynamical aspect.

Some authors (Chodos and Detweiler, 1980; Freund, 1982; Dereli and Tucker, 1983; Randjbar-Daemi *et al.*, 1984; Abbott *et al.*, 1987; Maeda, 1984; Sahadev, 1984) have obtained cosmological solutions of Einstein's equations in which the internal manifold contracts and 3-dimensional space expands with time. This note offers a compactification scheme based on the

[1]Department of Mathematics, North Eastern Hill University, Bijni Campus, Bhagyakul, Shillong 793003, India.

0020-7748/92/1200-2103$06.50/0

dynamics of a higher-dimensional model of the universe. Moreover, attention also is paid to the cosmological constant problem. As is well known, the energy density due to the cosmological constant is extremely small today, even less than 10^{-46} GeV^4. But if one believes general relativity at the Planck scale, one gets the value as $\sim 10^{76}$ GeV^4. The problem is to understand how the value of the energy density due to the cosmological constant has come down by more than 122 decimal places today. In this note, through dimensional reduction of the $(4+D)$-dimensional gravity (without cosmological constant), an effective 4-dimensional action for gravity with a cosmological constant term is obtained. The 4-dimensional cosmological constant thus obtained is time-dependent and asymptotically decreases with time. Thus a possible solution of the cosmological constant problem is suggested here. The natural units $\hbar = c = 1$ are used, where $\hbar$ and c have their usual meaning.

We consider a $(4+D)$-dimensional space, with coordinates that can be separated into the coordinates x^μ of ordinary space-time plus the coordinates y^a of the D-dimensional internal manifold. The line element is the generalized Robertson–Walker line element for $(4+D)$ dimensions, which is given as

$$ds^2 = -dt^2 + r^2(t)\delta_{ij}\, dx^i\, dx^j + R^2(t) g_{ab}(y)\, dy^a\, dy^b \tag{1}$$

where $i, j = 1, 2, 3$ and $a, b = 4, \ldots, (D+3)$; $r(t)$ and $R(t)$ are scale factors for M^3 and S^D, respectively; t is the cosmic time; δ_{ij} is the 3×3 unit matrix; and $g_{ab}(y)$ is the metric tensor on S^D. Since S^D is a compact manifold, it can be either homeomorphic to a sphere or to a connected sum of tori or to a connected sum of real projective planes (Massey, 1967). For simplicity here, S^D is taken to be a D-dimensional sphere. Hence

$$\begin{aligned} &g_{ab}(y)\, dy^a\, dy^b \\ &\quad = \rho^2(d\theta_1^2 + \sin^2\theta_1\, d\theta_2^2 + \cdots + \sin^2\theta_1 \cdots \sin^2\theta_{D-1}\, d\theta_D^2) \end{aligned} \tag{2}$$

where ρ is the physical radius of S^D and $\theta_1, \theta_2, \ldots, \theta_D$ are angular coordinates.

The energy-momentum tensor for a perfect fluid can be written

$$T_{MN} = (\varepsilon + p) u_M u_N + (\delta p + \delta' P) g_{MN} \tag{3a}$$

where ε is the energy density, p is the isotropic pressure on M^3, P is the isotropic pressure on S^D, and $u^M u_M = -1$, with $u^0 = 1$,

$$u^1 = u^2 = \cdots = u^{D+3} = 0$$

Also, $\delta' = 1 - \delta$, where

$$\delta = \begin{cases} 1 & \text{for} \quad M, N = 0, 1, 2, 3 \\ 0 & \text{for} \quad M, N = 4, 5, \ldots, (D+3) \end{cases}$$

Hence,

$$T_0^0 = \varepsilon, \qquad T_i^i = p \qquad (i = 1, 2, 3)$$

and

$$T_m^m = P, \qquad m = 4, 5, \ldots, (D+3) \tag{3b}$$

Conservation of energy-momentum yields

$$\dot{\varepsilon} + \varepsilon\left(3\frac{\dot{r}}{r} + D\frac{\dot{R}}{R}\right) + 3p\frac{\dot{r}}{r} + DP\frac{\dot{R}}{R} = 0 \tag{3c}$$

The matter field considered here is noninteracting as well as conformally invariant. So the energy-momentum tensor will be of the same form as (3a) and its trace will vanish. For convenience p and P are taken equal, so

$$\varepsilon = (D+3)p \tag{4a}$$

The conservation equation (3c) now yields

$$\dot{\varepsilon} + \left(3\frac{\dot{r}}{r} + D\frac{\dot{R}}{R}\right)(\varepsilon + p) = 0 \tag{4b}$$

where the overdot denotes $\partial/\partial t$. Using (4a), one can integrate (4b), giving

$$\varepsilon = \frac{(D+3)M}{(r^3 R^D)^{(D+4)/(D+3)}} \tag{5}$$

where M is an arbitrary integration constant.

In terms of the dimensionless parameter $\tilde{t} = t/t_P$ (t_P is the Planck time), Einstein's field equations are

$$3\frac{r''}{r} + D\frac{R''}{R} = -\frac{8\pi\bar{G}(D+3)Mt_P^2}{[r^3(\tilde{t})R^D(\tilde{t})]^{(D+4)/(D+3)}} \tag{6a}$$

$$\frac{d}{d\tilde{t}}\left(\frac{r'}{r}\right) + \frac{r'}{r}\left(3\frac{r'}{r} + D\frac{R'}{R}\right) = \frac{8\pi\bar{G}Mt_P^2}{[r^3(\tilde{t})R^D(\tilde{t})]^{(D+4)/(D+3)}} \tag{6b}$$

$$\frac{(D-1)R_D t_P^2}{\rho^2 R^2} + \frac{d}{d\tilde{t}}\left(\frac{R'}{R}\right) + \frac{R'}{R}\left(3\frac{r'}{r} + D\frac{R'}{R}\right) = \frac{8\pi\bar{G}Mt_P^2}{[r^3(\tilde{t})R^D(\tilde{t})]^{(D+4)/(D+3)}} \tag{6c}$$

where the prime denotes $\partial/\partial\tilde{t}$ and the $(4+D)$-dimensional gravitational constant is given by

$$\bar{G}=\frac{2\pi^{(D+1)/2}}{\Gamma\frac{1}{2}(D+1)}\rho^{D}G_{N} \tag{7}$$

with G_N the Newtonian gravitational constant and $\Gamma\kappa$ is the gamma function of κ.

Introducing $r^3R^D=A^{D+3}[\sigma(\tilde{t})]^{D+3}$ with $A^{D+4}=8\pi\bar{G}Mt_P^2$, one gets one more differential equation:

$$3\frac{r'}{r}+D\frac{R'}{R}=\frac{[\sigma^{(D+3)}]'}{\sigma^{(D+3)}} \tag{8}$$

Using (6b) in (8) and integrating, one gets

$$\left(\frac{r'}{r}\right)\sigma^{D+3}=k+\tau \tag{9}$$

where k is an integration constant and τ is defined as

$$\tau=\int^{\tilde{t}}\frac{d\tilde{t}_1}{\sigma(\tilde{t}_1)} \tag{10}$$

Differentiating (8) with respect to $\tilde{t}$ and using (9) and (6a), one gets a nonlinear second-order differential equation for σ:

$$\begin{aligned}\frac{D+3}{\sigma}\frac{d^2\sigma}{d\tau^2}-\frac{2(D+3)}{\sigma^2}\left(\frac{d\sigma}{d\tau}\right)^2+\frac{D+3}{\sigma^{D+2}}+\frac{3(k+\tau)^2}{\sigma^{2(D+2)}}&\\ +\frac{1}{D}\left[\frac{D+3}{\sigma}\frac{d\sigma}{d\tau}-\frac{3(k+\tau)}{\sigma^{D+2}}\right]^2=0&\end{aligned} \tag{11}$$

The equation admits the solution

$$\sigma^{D+2}=\frac{D+2}{2}(k+\tau)^2 \tag{12}$$

with the boundary condition $\sigma=0$ at $\tau=-k$.

Using (10) in (12) and integrating gives

$$\tilde{t}=\left(\frac{D+2}{2}\right)^{1/(D+2)}\left(\frac{D+2}{D+4}\right)(k+\tau)^{(D+4)/(D+2)} \tag{13}$$

with the condition that $\tilde{t}=0$ when $\tau=-k$. Equation (9) yields the solution

$$r=r_0\left[\left(\frac{2}{D+2}\right)^{1/(D+2)}\left(\frac{D+4}{D+2}\right)\tilde{t}\right]^{2/(D+4)} \tag{14}$$

Multiplying (6c) by D and subtracting it from (6a), we find that

$$D\left(\frac{R'}{R}\right)^2-(D+3)\frac{\sigma'}{\sigma}\frac{R'}{R}+3\frac{r''}{r}+\frac{2D+3}{\sigma^{D+4}}$$
$$-\frac{D(D-1)k_D r^{6/D}t_P^2}{A^{D+3}\rho^2\sigma^{2(D+3)/D}}=0 \tag{15}$$

Using (12)–(14) in (15) yields

$$\left(\frac{R'}{R}\right)^2-\frac{(D+3)(D+2)}{D(D+4)\tilde{t}}\frac{R'}{R}+\frac{4(D+2)}{(D+4)^2(\tilde{t})^2}-\frac{\beta^2}{(\tilde{t})^{4/(D+4)}}=0 \tag{16}$$

where

$$\beta^2=\frac{D(D-1)R_D t_P^2}{\rho^2}\left(\frac{R_0^3}{A^{D+3}}\right)^{2/D}\left(\frac{D+2}{D+4}\right)^{4/(D+4)}$$
$$\times\left(\frac{2}{D+2}\right)^{2(-D^2-8D+12)/D(D+2)(D+4)} \tag{17}$$

From (16) one gets

$$\frac{R'}{R}=\frac{1}{2}\left(\frac{(D+3)(D+2)}{D(D+4)\tilde{t}}\right.$$
$$\left.\pm\left\{\left[\frac{(D+3)(D+2)}{D(D+4)\tilde{t}}\right]^2-\frac{16(D+2)}{(D+4)^2(\tilde{t})^2}+\frac{4\beta^2}{(\tilde{t})^{4/(D+4)}}\right\}^{1/2}\right) \tag{18}$$

If $R(t)$ acquires maxima at $\tilde{t}=\tilde{t}_m$, then from (18)

$$\frac{(D+3)(D+2)}{D(D+4)\tilde{t}_m}$$
$$\pm\left\{\left[\frac{(D+3)(D+2)}{D(D+4)\tilde{t}_m}\right]^2-\frac{16(D+2)}{(D+4)^2(\tilde{t}_m)^2}+\frac{4\beta^2}{(\tilde{t}_m)^{4/(D+4)}}\right\}^{1/2}=0 \tag{19}$$

If the condition of maxima is used in (16), then

$$\frac{\beta^2}{(\tilde{t}_m)^{4/(D+4)}}=\frac{4(D+2)}{(D+4)^2(\tilde{t}_m)^2} \tag{20}$$

Looking at (19) and (20), one should take only the negative square root in (18), for the sake of consistency. Moreover, from (17) and (20), one gets

$$A^{2(D+3)/D} = \frac{D(D-1)k_D t_P^2 (D+4)^2 (\tilde{t}_m)^{(2D+4)/(D+4)}}{4\rho^2 (D+2)^{D/(D+4)} (D+4)^{4/(D+4)}} \times r_0^{6/D} \left(\frac{2}{D+2}\right)^{2(-D^2-8D+12)/D(D+2)(D+4)}$$

This result shows that for $D=1$, $A=0$, which leads to an inconsistency, as it yields $r=0$, $R=0$, or $M=0$. This means that the results are not true for $D=1$. So, we will not consider the $D=1$ case henceforth.

When $\tilde{t}<1$, (18) can be approximated as

$$\frac{R'}{R} \simeq \frac{(D+3)(D+2)}{2D(D+4)\tilde{t}} - \left[\frac{D+2}{2D(D+4)}\left(\frac{D^3-8D^2+21D+18}{D+2}\right)^{1/2}\right]\frac{1}{\tilde{t}} \tag{21}$$

which yields the solution

$$R(t) = \bar{R}_0 (\tilde{t})^{[(D+2)/2D(D+4)]\{(D+3)-[(D^3-8D^2+21D+18)/(D+2)]^{1/2}\}} \tag{22}$$

where $\bar{R}_0$ is an integration constant.

When $\tilde{t}>1$, (18) is approximated as

$$\frac{R'}{R} \simeq \frac{(D+3)(D+2)}{2D(D+4)\tilde{t}} - \frac{\beta}{(\tilde{t})^{2/(D+4)}} \tag{23}$$

which is integrated to

$$R(t) = R_0 (\tilde{t})^{(D+3)(D+2)/2D(D+4)} \exp\left[-\beta \frac{D+4}{D+2} (\tilde{t})^{(D+2)/(D+4)}\right] \tag{24}$$

where R_0 is an integration constant.

From (14), (22), and (24), one learns that $R(t)$ starts from $R(0)=0$ like $r(t)$ and both expand until $\tilde{t}<1$ ($t<t_P$). But when $\tilde{t}>1$ ($t>t_P$), $R(t)$ turns over and starts to collapse, while $r(t)$ still expands. Thus it is reasonable to infer that $R(t)$ has a maximum around $\tilde{t}\simeq 1$, i.e., $\tilde{t}_m \simeq 1$. Now, from (20)

$$\beta^2 = \frac{4(D+2)}{(D+4)^2} \tag{25}$$

At this juncture, one may ask, "Why does $R(t)$ start to collapse after an initial expansion?" One can answer this question by looking at (21) and (23). When $\tilde{t}<1$ ($t<t_P$), there is no term on the rhs of (21) which can make $R(t)$ decrease. But when $\tilde{t}>1$ ($t>t_P$) on the rhs of (23), there exists a term $-\beta/(\tilde{t})^{2/(D+4)}$ which is responsible for collapse, as is transparent from the solution (24). Also one can see that collapse is not possible if β vanishes. From (20), one finds that the possibility of β vanishing exists only when $D=\infty$. But $D=\infty$ is not feasible. Realistic higher-dimensional theories are described in finite-dimensional spaces. For example, a realistic model of $N=1$ supergravity theory exists in 11 dimensions, bosonic strings are described in 26 dimensions, and fermionic strings as well as superstrings are possible in 10 dimensions. Moreover, compactification of the model considered is not possible if $R(t)$ does not turn over.

One can also notice that when $\tilde{t}>1$, the solution (24) shows that $R(t)$ does not stabilize itself at an early epoch. It stabilizes at $R=0$ when $t=\infty$, though (as discussed later) $\rho R(t)\lesssim L_P$ (Planck length) when $t\gtrsim t_c$ (compactification time). Thus $R(t)$ has a "crack of doom" singularity when $t\to\infty$.

Thus we find that compactification of the model is possible when $\tilde{t}>1$ ($t>t_P$). The effective radius for S^D is $\rho R(t)$. When $\tilde{t}>1$, it is given by (24) and (25) as

$$\rho R(t)=\rho R_0 f(t) \tag{26}$$

where

$$f(t)=(\tilde{t})^{(D+3)(D+2)/2D(D+4)}\exp\left[-\frac{2}{(D+2)^{1/2}}(\tilde{t})^{(D+2)/(D+4)}\right] \tag{27}$$

ρ and R_0 are both arbitrary, so it is assumed that

$$\rho R_0=L_P \quad \text{(Planck length)} \tag{28}$$

Under this assumption, which does not harm the physics, the effective radius is equal to $L_P f(t)$. Thus the effective radius of the compact manifold S^D increases or decreases as $f(t)$. Our observable universe is 4-dimensional. This means that the extra D-dimensional space is hidden, being extremely small (undetectable) in size, around L_P, the Planck length (Toms, 1986). So it is expected that the effective radius of S^D should not be greater than L_P at the time when we get an approximately 4-dimensional model of the universe (Kolb and Slansky, 1984). This particular time is called the compactification time t_c. Based on this idea, it is inferred that

$$f(t_c)\lesssim 1 \tag{29}$$

Connecting (27) and (29) yields

$$(\tilde{t}_c)^{(D+3)(D+2)/2D(D+4)}\exp\left[-\frac{2}{(D+2)^{1/2}}(\tilde{t}_c)^{(D+2)/(D+4)}\right]\lesssim 1 \tag{30}$$

On taking the logarithm of both sides of (30), we find

$$\frac{D+3}{2D}\ln(\tilde{t}_c)^{(D+2)/(D+4)} \lesssim \frac{2}{(D+2)^{1/2}}(\tilde{t}_c)^{(D+2)/(D+4)}$$

which yields, on expanding the logarithm and neglecting higher-order terms of the left-hand side,

$$\tilde{t}_c \lesssim \left[\frac{1}{1-4D/(D+3)(D+2)^{1/2}}\right]^{(D+4)/(D+2)} \tag{31}$$

Using (31), one can compute $\tilde{t}_c$ for different values of D. The compactification mass scale M_c can also be calculated by inverting $\tilde{t}_c$ and multiplying it by M_P (Planck mass). This formula is not valid for $D=1$, as it is noted above that compactification of the internal manifold is not possible for $D=1$ in this model. In the Table I, $\tilde{t}_c = t_c/t_P$ and $\tilde{M}_c = (\tilde{t}_c)^{-1} = M_c/M_P$ are given for different values of D. It is noted that $\tilde{t}_c/\tilde{M}_c$ increases (decreases) up to $D=5$ but decreases (increases) from $D=6$.

The action for gravity in the cosmological model under consideration can be written as

$$\bar{S}_g = -\frac{1}{16\pi\bar{G}}\int dt\, d^3x\, d^Dy\, r^3 R^D [g(y)]^{1/2} \mathscr{R}_{4+D} \tag{32}$$

where the bar over S_g and G denotes these quantities in $(4+D)$-dimensional space-time, $\mathscr{R}_{4+D}$ is the $(4+D)$-dimensional Ricci scalar, and $[g(y)]^{1/2}$ is the square root of the determinant of the metric tensor on S^D.

Under the conformal transformation

$$g_{MN} \to g^*_{MN} = \Omega^{-2}(t) g_{MN}$$

Table I. Values of $\tilde{t}_c/\tilde{M}_c$ for Different D, from Equation (31)

D	$\tilde{t}_c = t_c/t_P$	$\tilde{M}_c = M_c/M_P$
2	11.18	0.089
3	23.28	0.043
4	36.8	0.027
5	41.56	0.024
6	35.76	0.028
7	27.38	0.037
8	20.695	0.048
9	16.056	0.062
10	12.891	0.078

$\bar{S}_g$ given by (32) can be written as

$$\bar{S}_g = -\frac{1}{16\pi\bar{G}}\int dt\, d^3x\, d^Dy\, \Omega^{2+D}(-g^*_{4+D})^{1/2}$$
$$\times[\mathscr{R}^*_4 + \mathscr{R}^*_D + 2(D+3)\Omega^{-1}\Omega_{;MN}g^{*MN}$$
$$+D(D+3)\Omega^{-2}\Omega_{,M}\Omega_{,N}g^{*MN}] \tag{33a}$$

where

$$\mathscr{R}^*_D = -D\Omega^2\left\{\frac{(D-1)R_D}{\rho^2R^2} + \frac{d}{dt}\left(-\frac{\dot{\Omega}}{\Omega}+\frac{\dot{R}}{R}\right) + \frac{\dot{\Omega}}{\Omega}\left(-\frac{\dot{\Omega}}{\Omega}+\frac{\dot{R}}{R}\right)\right.$$
$$\left. + \left(-\frac{\dot{\Omega}}{\Omega}+\frac{\dot{R}}{R}\right)\left[3\left(-\frac{\dot{\Omega}}{\Omega}+\frac{\dot{R}}{R}\right) + D\left(-\frac{\dot{\Omega}}{\Omega}+\frac{\dot{R}}{R}\right)\right]\right\} \tag{33b}$$

and

$$(-g^*_{4+D})^{1/2} = \Omega^{-(4+D)}r^3R^D[g(y)]^{1/2}$$

Now choosing $\Omega^2 = R^D$ and performing integration over y with

$$\int d^Dy[g(y)]^{1/2} = \frac{2\pi^{(D+1)/2}}{\Gamma(D+1)/2}\rho^D$$

One gets, from (33), the effective 4-dimensional Einstein–Hilbert action for gravity as

$$S^{(4)}_g = -\frac{1}{16\pi G_N}\int d\tau\, d^3x\, r^3R^{-3D/2}[\mathscr{R}^*_4 - 2\Lambda(t)] \tag{34a}$$

where $\tau = \int^t R^{3D/2}(t')\,dt'$, $\mathscr{R}^*_4$ is the Ricci scalar in 4-dimensional effective gravity, the 4-dimensional metric tensor is

$$\mathrm{diag}(1, -r^2(\tau)R^{-D}(\tau), -r^2(\tau)R^{-D}(\tau), -r^2(\tau)R^{-D}(\tau))$$

and

$$2\Lambda(t) = -\mathscr{R}^*_D - 2(D+3)\Omega\Omega_{;MN}g^{MN} - D(D+3)\Omega_{,M}\Omega_{,N}g^{MN}$$
$$= \frac{DR^D}{t_P^2}\left[\frac{(D-1)k_DR_0^2t_P^2}{L_P^2R^2} - \frac{1}{2}(3D+4)\frac{d}{d\tilde{t}}\left(\frac{R'}{R}\right)\right.$$
$$\left. - \frac{D(D+2)(3D-D^2+3)}{4}\left(\frac{R'}{R}\right)^2 + 3\left(1-\frac{D}{2}\right)\frac{R'}{R}\frac{r'}{r}\right] \tag{34b}$$

It is interesting to see that the dimension of the extra manifold has a significant impact in the effective 4-dimensional gravity. Also, time is redefined.

The $\Lambda(t)$ given by (34b) is the term induced by the internal manifold which contributes to the energy-momentum tensor of the vacuum (in the 4-dimensional theory). So $\Lambda(t)$ can be identified with cosmological constant (Weinberg, 1989), which is time-dependent.

When $\tilde{t}<1$ ($t<t_P$), using (14) and (22), one gets from (34b)

$$\Lambda(t)=\frac{D(\bar{R}_0)^D(\tilde{t})^{D\alpha}}{2t_P^2}\left\{\frac{(D-1)k_D R_0^2}{(\bar{R}_0)^2(\tilde{t})^{2\alpha}}\right.$$
$$\left.+\left[\frac{(3D+4)\alpha}{2}-\frac{D(D+2)(3D-D^2+3)\alpha^2}{4}+\frac{6(1-D/2)\alpha}{D+4}\right](\tilde{t})^{-2}\right\} \tag{35}$$

where

$$\alpha=\frac{D+2}{2D(D+4)}\left[(D+3)-\left(\frac{D^3-8D^2+21D+18}{D+2}\right)^{1/2}\right]$$

When $\tilde{t}>1$ ($t>t_P$), using (14) and (24), one gets

$$\Lambda(t)=\frac{DR_0^D(\tilde{t})^{(D+3)(D+2)/2(D+4)}}{2t_P^2}$$
$$\times\exp\left[\frac{\beta D(D+4)}{D+2}(\tilde{t})^{(D+2)/(D+4)}\right]$$
$$\times\left\{\frac{(D-1)k_D t_P^2}{L_P^2}(\tilde{t})^{-(D+3)(D+2)/D(D+4)}\right.$$
$$\times\exp\left[\frac{2\beta(D+4)}{(D+2)}(\tilde{t})^{(D+2)/(D+4)}+\frac{1}{2}(3D+4)\right]$$
$$\times\left[\frac{(D+3)(D+2)}{2D(D+4)(\tilde{t})^2}-\frac{2\beta}{D+4}(\tilde{t})^{-(D+6)/(D+4)}\right]$$
$$-\frac{D(D+2)(3D-D^2+3)}{4}\left[\frac{(D+3)(D+2)}{2D(D+4)\tilde{t}}-\frac{\beta}{(\tilde{t})^{2/D+4}}\right]^2$$
$$\left.+\frac{6}{D+4}(1-D/2)\left[\frac{(D+3)(D+2)}{2D(D+4)(\tilde{t})^2}-\frac{\beta}{(\tilde{t})^{(D+6)/(D+4)}}\right]\right\}$$

From (35) and (36), one gets that for $D=2$, $\Lambda(t)>M_P^2$ when $t<t_P$ and $\Lambda(t)\sim M_P^2$ when $t>t_P$. When $D\geq 3$, Λ is still greater than M_P^2 for $t<t_P$, but it decreases rapidly for $t>t_P$. Thus (36) implies that at late times $\Lambda(t)\approx 0$.

REFERENCES

Abbott, R. B., Barr, S. M., and Ellis, S. D. (1987). *Physical Review D*, **30**, 720.
Appelquist, T., Chodos, A., and Freund, P. G. O. (1987). *Modern Kaluza–Klein Theories*, Addison-Wesley, Reading, Massachusetts.
Carter, B. (1977). *Physical Review D*, **16**, 3395.
Chodos, A., and Detweiler, S. (1980). *Physical Review D*, **21**, 2167.
Dereli, T., and Tucker, R. W. (1983). *Physics Letters*, **125B**, 133.
Freund, P. G. O. (1982). *Nuclear Physics B*, **209**, 146.
Kaluza, T. (1921). *Sitzungberichte der Preussichen Akademie der Wissenschaften zu Berlin, Mathematik Physik Klasse*, **1921**, 966.
Klein, O. (1926). *Zeitschrift für Physik*, **37**, 89.
Kolb, E. W., and Slansky, R. (1984). *Physics Letters*, **145B**, 39.
Maeda, K. (1984). *Physical Review D*, **30**, 2482.
Massey, W. S. (1967). *Algebraic Topology: An Introduction*, Harcourt, Brace and World.
Miller, Jr., W. (1977). *Symmetry and Separation of Variables*, Addison-Wesley, Reading, Massachusetts.
Randjbar-Daemi, Salam, A., Strathdee, J. (1984). *Physics Letters*, **135B**, 388.
Sahadev, D. (1984). *Physical Review D*, **30**, 2495.
Toms, D. J. (1986). *Canadian Journal of Physics*, **64**, 644.
Weinberg, S. (1989). *Review of Modern Physics*, **61**, 1.